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Bachelors of Science

University of New Brunswick (2025)
Masters of Science

Interpolating between splittings over cyclic subgroups

UNIVERSITY OF NEW BRUNSWICK

THESIS DEFENCE AND EXAMINATION

in Partial Fulfillment

of the Requirement for the Degree of
Master of Science

by

Benjamin R. Cook

in the Department of Mathematics & Statistics

U.N.B., Fredericton, N.B.

Thursday, March 20th, 2025
3:00 p.m.

Tilley Hall, Room 104

Examining Committee

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Abstract

Given a finitely generated and torsion-free group G, it may be possible to express G as a product of smaller groups. In particular, we consider graph of groups decompositions of G — henceforth called *splittings* — wherein G is expressed as a collection of groups that are amalgamated over shared subgroups, and the underlying structure presented as a graph. This can be understood as an analogue of prime factorization in the natural numbers, except that G may not have a unique factorization. In this thesis, we present a universal process for interpolating between two distinct splittings **X** and **Y** of a group G to obtain a third splitting **Z**, also of the group G. That such an interpolation is possible was shown by Fujiwara and Papasoglu in 2006; we provide similar results in a more geometric fashion. Our process is predicated on the notion of duality introduced by Jean-Pierre Serre, allowing us to take splittings **X** and **Y** of G and express them as group actions of G on trees X and Y, which we may then manipulate to obtain a third tree Z. Serre's results allow us to then translate this tree into a dual splitting **Z**. While we provide this result in the general case, we end the thesis by specifically considering cyclic one-edge cases — where the graph of groups has only one edge, representing a cyclic subgroup — and the three possible combinations of such one-edge splittings. Each of these three combinations yields a unique structure for the resulting interpolated splitting **Z**.